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What symmetry is broken in the superconductor–normal phase transition?

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Abstract. We show that the superconducting–normal phase transition is due to spontaneous breaking of magnetic flux symmetry. In two dimensions the symmetry generator is $\Phi = \int d^2x\, B(x)$ and in three dimensions there are three generators $\Phi_i = \int d^3x\, B_i(x)$, only two of which are independent due to the absence of sources of magnetic field. In the normal phase the symmetry is spontaneously broken with a massless photon as a corresponding Goldstone boson. In the superconducting phase the symmetry is unbroken and the magnetic flux annihilates the vacuum which expresses the essence of the Meissner effect. In two dimensions we explicitly construct the pertinent gauge-invariant order parameter which is the operator creating Abrikosov vortices. Its vacuum expectation value vanishes in the superconducting ground state, while it is finite in the vacuum of the normal phase.

1. Introduction

Second-order phase transitions are frequently, if not always, associated with spontaneous breakdown of a global symmetry. It is then possible to find a corresponding order parameter which vanishes in the disordered phase and is non-zero in the ordered phase. Qualitatively the transition is understood as condensation of the broken symmetry 'charge' carriers. The critical region is effectively described by a local Lagrangian involving the order parameter field.

At the first glance the superconductor–normal phase transition is an important exception. Colloquially it is said sometimes that the electric charge $U_e(1)$ symmetry is broken in superconductors with a non-vanishing 'order parameter' $\Delta \equiv \langle\psi_\uparrow\psi_\downarrow\rangle$. A little thought, however, shows that this should not be taken literally. If a continuous symmetry is spontaneously broken, the Goldstone theorem [1] ensures the existence of massless excitations. For example, in planar or Heisenberg (anti)ferromagnets one always finds soft magnons. On the other hand, the physical spectrum of a superconductor does not contain any such excitation. Also the ' order parameter' Δ does not vanish *only when there is no coupling to electromagnetism*. Consequently in the complete theory including electromagnetism, $U_e(1)$ is not broken spontaneously. The interpretation of this is that the 'would-be Goldstone boson' is eaten by the photon

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which acquires a mass (finite penetration depth) via the Anderson–Higgs mechanism [1].

This mechanism ('local gauge symmetry breaking'), however, lacks description in physical terms. Indeed, according to the very general Elitzur's theorem [2], local symmetry can never be broken and a non-gauge-invariant quantity never acquires a non-zero vacuum expectation value (VEV).

Thus neither global electric charge $U_e(1)$ nor local gauge symmetry is broken at the superconductor–normal phase transition. However, as we show in this paper, such a symmetry does exist. It is identified here as a continuous symmetry generated by a magnetic flux. Here we outline the definition of the symmetry and the ensuing physical picture of the phase transition in the simpler two-dimensional case. The more complicated three-dimensional case is deferred to section 4.

In two dimensions the flux symmetry is generated by the full magnetic flux penetrating the plane

$$\Phi = \int B(x)\,\mathrm{d}^2x. \tag{1}$$

The continuity equation for the corresponding current

$$\rho_\Phi \equiv B \qquad j^i_\Phi \equiv -\epsilon^{ij} E_j \qquad i, j, = 1, 2 \tag{2}$$

is the homogeneous Maxwell equations of electrodynamics†

$$\frac{1}{c}\frac{\partial B}{\partial t} = -\epsilon_{ij}\partial_i E_j. \tag{3}$$

We show that the mode of realization of this $U_\Phi(1)$ symmetry distinguishes between the superconducting and the normal phases. In the normal phase $U_\Phi(1)$ is spontaneously broken. The Goldstone theorem then requires existence of massless mode. It is shown in section 2 that this is the (massless‡) photon. The superconducting vacuum is invariant under the action of $U_\Phi(1)$, the symmetry is unbroken and indeed there are no massless modes in the superconducting state.

In section 2 we construct the order parameter $V(x)$, which is generally an eigenoperator of Φ

$$[\Phi, V(x)] = -gV(x). \tag{4}$$

In the superconductor the VEV of $V(x)$ vanishes and it creates the Abrikosov vortex carrying g units of magnetic flux. In the normal phase $V(x)$ acquires non-zero VEV which distinguishes between different degenerate vacua.

In the superconducting phase a single Abrikosov vortex is an eigenstate of the conserved 'charge' Φ. The energy gap Δ_Φ between the vacuum and the lightest flux carrying state 'the vortex' is finite. This expresses the essence of the Meissner effect. Application of small external magnetic field amounts to the addition to the

† Introduction of vector potential A_i solves this equation explicitly. This does not mean that the equation is void of dynamical content.

‡ The phase we term normal can be either metal or insulator. Masslessness of photon is understood as the absence of magnetic mass which is relevant for superconductivity. In the metallic state a non-zero electric mass is present due to plasma oscillations.

Hamiltonian of a perturbation ΔH, which also commutes with Φ. The perturbed vacuum acquires an admixture of excited states, but since ΔH commutes with Φ it cannot contain states with non-zero flux. Consequently infinitesimal magnetic fields cannot penetrate the sample. However when the external field is large enough this perturbative argument is no longer valid, and eventually a fluxon is produced. The required magnetic energy must be larger than the energy gap in the fluxon channel. Therefore H_{c_1} is determined by Δ_Φ. In this picture it is clear that the creation of fluxons is the only way the magnetic field can penetrate the sample since any mixed state should be an eigenstate of Φ.

We stress that in terms of symmetries of the *physical* Hilbert space the symmetry breaking pattern is reversed compared with the aforementioned colloquial terminology: in the superconducting phase no symmetry is broken while the normal state breaks the flux symmetry.

The paper is organized as follows. In section 2 we define the order parameter V in two dimensions and calculate its VEV in the normal phase. In section 3 we construct explicitly the vacuum wavefunctional in the superconducting phase and show that it is annihilated by Φ, and leads to vanishing VEV of V. In section 4 we outline the generalization to the three-dimensional case and give a short summary of the results. The appendix contains the dual representation of pure two-and three-dimensional electromagnetism in which the flux symmetry acts as a simple global shift of a massless field.

2. The two-dimensional flux symmetry and order parameter in the normal phase

Here we discuss in more detail the flux symmetry transformations in two dimensions. Consider the BCS-type Hamiltonian with two-body interaction U and chemical potential μ

$$H = \int \frac{1}{2m}\left[\psi_a^\dagger\left(\partial_i - \mathrm{i}\frac{e}{c}A_i\right)^2\psi_a + \mu\psi_a^\dagger\psi_a\right] + \int_{xy}\psi_a^\dagger(x)\psi_a(x)U(x-y)\psi_a^\dagger(y)\psi_a(y) + \tfrac{1}{2}\int[E_i^2 + B^2]. \tag{5}$$

In the Hamiltonian formalism in which

$$E_i = -\frac{1}{c}\frac{\partial A_i}{\partial t} \qquad B = \epsilon_{ij}\partial_i A_j$$

the following canonical commutation relations are imposed

$$[E_i(x), A_j(y)] = -i\delta_{ij}\delta^2(x-y)$$
$$\{\psi^\dagger(x), \psi(y)\} = \delta^2(x-y). \tag{6}$$

The flux 'charge' Φ is a gauge-invariant operator which commutes with A_i $\psi^\dagger$ and ψ. It also commutes with $E_i(x)$ for x not on the sample boundary. For x on the boundary

$$[\Phi, E_i(x)] = \frac{\mathrm{i}}{L}n_i(x) \tag{7}$$

where $n_i(x)$ is the unit tangent vector to the boundary at the point x and L is a linear dimension of the sample.

The order parameter corresponding to the flux symmetry breaking is a local eigenoperator of Φ, equation (1). Since Φ is linear in A_i, the operator V will be sought in the following form

$$V(x) = C \exp \mathrm{i}g \int \mathrm{d}^2 y \left[a_i(x-y)\hat{E}_i(y) + \frac{e}{c} b(x-y)\hat{n}(y) \right] \tag{8}$$

where $\hat{n}(x) \equiv \psi_a^\dagger \psi_a$. Equation (4) is satisfied by

$$\epsilon_{ij}\partial_i a_j(x) = \delta^2(x).$$

The requirement of locality of V with respect to gauge-invariant quantities $J_i \equiv \psi_a^\dagger(i\partial_i + eA_i)\psi_a$ further restricts the choice of $a(x)$ and $b(x)$. In particular, for $x \neq y$, the commutator

$$[J_i(x), V(y)] = \frac{2\mathrm{i}}{c}\{-ega_i(x-y) + \partial_i^x [egb(x-y)]_{\mathrm{mod}\,2\pi}\} n(x)V(y) \tag{9}$$

should vanish. There is no continuous function $b(x)$ for which the right-hand side of equation (9) vanishes. There are, however, discontinuous functions of this kind:

$$b(x) = \int_{C(x)} a_i \,\mathrm{d}l_i \tag{10}$$

where the contour $C(x)$ starts at the origin and ends at the point x. The function $b(x)$ has a branch cut which starts at the origin and depends on the choice of the contour $C(x)$. The simplest choice is

$$a_i(x) = \frac{1}{2\pi}\epsilon_{ij}\frac{x_j}{x^2} \qquad b(x) = \frac{1}{2\pi}\Theta(x) \tag{11}$$

where $\Theta(x)$ is an angle between the vector x_i and the $\hat{x}_1$ axis, $0 \leqslant \Theta < 2\pi$. In this case the discontinuity lies along the positive direction of the first axis. In order that the discontinuity of $b(x)$ does not spoil the locality of $V(x)$, the eigenvalue g must be quantized in units of $2\pi c/e$†

$$g = 2\pi cn/e. \tag{12}$$

This coincides with the usual quantization of magnetic flux of the Abrikosov vortices. The fact that V creates the vortex is seen from the relation

$$V(x)B(y)V^\dagger(x) = B(y) - g\delta^2(x-y). \tag{13}$$

The phase of the theory is determined by the VEV of the order parameter. We now calculate $\langle V \rangle$ in the normal phase and show that it does not vanish. Different values of the order parameter distinguish between degenerate vacua.

† The location of the discontinuity (the Dirac string) does not influence any gauge-invariant physical quantity. This is so, since the charge density oprator $n(x)$ is quantized and the discontinuity across the cut is a multiple of 2π and therefore is not felt in the exponent equation (8).

To the leading (zeroth) order in e the vacuum state in the normal phase is the direct product of the fermionic Fock vacuum $|\emptyset\rangle_{\rm f}$ and the vacuum photonic state $|\emptyset,\zeta\rangle_{\rm ph}$. The parameter ζ labels different vacua connected to the standard Fock vacuum $|\emptyset\rangle_{\rm ph} \equiv |\emptyset,\zeta=0\rangle_{\rm ph}$ by the transformation

$$|\emptyset,\zeta\rangle_{\rm ph} = {\rm e}^{{\rm i}\zeta\Phi}|\emptyset\rangle_{\rm ph}. \tag{14}$$

The fact that the flux Φ does not annihilate the state $|\emptyset\rangle_{\rm ph}$ is seen as follows. In the momentum space

$$\Phi = \lim_{k\to 0}\omega^{1/2}(k)(a^\dagger(k)+a(k)) \qquad \omega(k) = c\sqrt{k^2}. \tag{15}$$

This operator is not identically zero (although $\omega(0)=0$) since the normalization of creation and annihilation operators is not 1 but a Dirac delta function. With an infrared regulator L (the finite size of the sample) the commutator is

$$[a_k^\dagger, a_l] = L^2\delta_{k,-l}.$$

Applying Φ on the vacuum state one obtains a state with one soft photon

$$\Phi|\emptyset\rangle_{\rm ph} \propto L^{1/2}|\omega(1/L)\rangle. \tag{16}$$

This diverges in the limit $L\to\infty$ as is usually the case with matrix elements of spontaneously broken charge [3]. The vacua with different ζ are coherent states of the soft photons.

Another way to see this is to use the Schrödinger representation of the quantum field theory (QFT) [4] which will be useful for our purposes. The wavefunctional of the Fock vacuum in the field basis is the Gaussian

$$\Psi_0[A_i(x)] = N\exp\left\{-\frac{c^2}{2}\int\frac{{\rm d}^2k}{(2\pi)^2}A_i(k)\omega(k)\left(\delta_{ij}-\frac{k_ik_j}{k^2}\right)A_j(-k)\right\} \tag{17}$$

where N is the normalization factor. Note that Ψ_0 does not depend on the zero frequency component of A_i. Therefore the action of $A_i(0)$ on the vacuum leads to a non-normalizable wavefunctional† $A_i(0)\Psi_0[A_i(x)]$. Therefore we see again that Φ does not annihilate the Fock vacuum. The set of the degenerate vacua is given in this representation by

$$\langle A_i(x)|\emptyset,\zeta\rangle_{\rm ph} = \Psi_\zeta[A_i(x)] = \Psi_0[A_i(x)]{\rm e}^{{\rm i}\zeta\int{\rm d}^2x B(x)}. \tag{18}$$

There is yet another convenient way to see the degeneracy of the QED vacuum. In two dimensions the free photon is completely equivalent to free massless relativistic scalar by duality transformation. The flux transformation then is just the shift of the scalar field. This is described in detail in appendix.

† This is fully analogous to the quantum mechanics of a free particle. There the ground state is a constant $\psi_0(x) = N$ and $x\psi_0(x)$ is not normalizable.

We now calculate the expectation value of the order parameter V in the Fock vacuum. The straightforward calculation gives

$$\langle\emptyset|\exp\left[ig\int d^2x\, a_i(x)E_i(x)\right]|\emptyset\rangle = \exp\left[-\frac{g^2}{2}\left\langle\emptyset\left|\left(\int a_i E_i\right)^2\right|\emptyset\right\rangle\right]$$
$$= \exp\left[-\frac{g^2}{2c}\int\frac{d^2k}{4\pi^2}\frac{1}{|k|}\right] = exp\left[-g^2\frac{\Lambda}{2\pi c}\right] \tag{19}$$

where Λ is an ultraviolet cutoff. For other states

$$\langle\zeta|V|\zeta\rangle = \exp\left(\frac{-g^2}{2\pi c}\Lambda\right)\exp(ig\zeta).$$

The corrections to this result are small for small e and can be systematically calculated in perturbation theory [5]. The spontaneous breaking of the flux symmetry in the normal phase leads to the appearance of a massless Goldstone mode. This is clearly the massless photon. The density of the flux charge $B(x)$ is linear in the photon creation and annihilation operators and the flux charge itself creates from the vacuum the state with one soft phonon equations (15)–(16).

3. Superconducting phase

To perform the calculation in the superconducting state it is more convenient to use the low-energy effective Hamiltonian instead of the original BCS one, equation (5). In the superconducting state the relevant excitations can be described by the composite complex field $\Delta(x) \equiv \psi_\uparrow(x)\psi_\downarrow(x)$. Assuming that the absolute value of the Δ(x) is fixed

$$\Delta(x) = \Delta_0 e^{i\theta(x)}$$

and integrating over original fermions one obtains (see, for example, [6])

$$H = H_2 + \tilde{H}.$$

The operator $\tilde{H}$ contains terms with more than two derivatives and/or powers of π and

$$H_2 = \frac{1}{\Delta_0^2}\pi^2 + \mu\pi + v^2\Delta_0^2\left(\partial_i\theta - \frac{q}{c}A_i\right)^2 + \tfrac{1}{2}E^2 + \tfrac{1}{2}B^2. \tag{20}$$

Here π is a field canonically conjugate to θ

$$[\pi(x), \theta(y)] = -i\delta(x-y). \tag{21}$$

The charge q is twice that of an electron, $v^2 \equiv v_F^2/3$ and μ is the chemical potential†.

† The form of H_2 does not depend on the two-body interaction $U(x-y)$ in equation (5) as long as the system is in the superconducting state.

The Coulomb constraint in terms of these variables is

$$C = \partial_i E_i - \frac{q}{c}\pi + \rho_0 = 0 \tag{22}$$

where ρ_0 is the charge density of the homogeneous neutralizing background.

We now write down explicitly the superconducting ground state in the (functional) Schrödinger representation and demonstrate that it is annihillated by Φ and that the VEV of the order parameter V vanishes.

It is useful to define a new variable $\tilde{\pi}$ instead of π, eliminating the linear term in the Hamiltonian, equation (20):

$$\tilde{\pi} = \pi + \tfrac{1}{2}\mu\Delta_0. \tag{24}$$

The quadratic part of the Hamiltonian and the constraint then become

$$H_2 = \frac{1}{\Delta_0^2}\tilde{\pi}^2 + v^2\Delta_0^2\left(\partial_i\theta - \frac{q}{c}A_i\right)^2 + \tfrac{1}{2}E^2 + \tfrac{1}{2}B^2 \tag{25}$$

and

$$C = \partial_i E_i - \frac{q}{c}\tilde{\pi} + \tilde{\rho}_0 \qquad \tilde{\rho}_0 \equiv \rho_0 - \tfrac{1}{2}\mu\Delta_0. \tag{26}$$

Canonical commutation relations of $\tilde{\pi}$ with θ are identical to those of π.

We shall find the vacuum of H_2 assuming that higher derivative terms can be treated perturbatively.

In the Schrödinger functional representation the canonically conjugate momenta are represented by functional derivatives:

$$\tilde{\pi}(x) = -\mathrm{i}\frac{\delta}{\delta\theta(x)} \qquad E^i(x) = -\mathrm{i}\frac{\delta}{\delta A^i(x)}. \tag{27}$$

In the momentum space these become

$$\tilde{\pi}(k) = -\mathrm{i}\frac{\delta}{\delta\theta(-k)} \qquad E^i(k) = -\mathrm{i}\frac{\delta}{\delta A^i(-k)}. \tag{28}$$

Imposing the constraint equation (26) on the physical states in the Hilbert space leads to the following form of the admissible wavefunctionals

$$\Psi[\theta(x), A_i(x)] = \exp\left[\mathrm{i}\frac{c}{q}\tilde{\rho}\int \mathrm{d}^2x\,\theta(x)\right]\tilde{\Psi}\left[\partial_i\theta - \frac{q}{c}A_i, B\right] \tag{29}$$

where the wavefunctional $\tilde{\Psi}$ obeys the linear homogeneous constraint equation

$$\left(\partial_i E_i - \frac{q}{c}\tilde{\pi}\right)\tilde{\Psi} = 0. \tag{30}$$

The Hamiltonian H_2 is quadratic in the remaining variables and thus its ground state is a Gaussian wavefunctional

$$\begin{aligned}\tilde{\Psi} = N\exp\{-\tfrac{1}{2}[&\theta(k)G(k)\theta(-k) + A_i(k)G_{ij}(k)A_j(-k) \\ &+ 2\theta(k)G_i(k)A_i(-k)]\}.\end{aligned} \tag{31}$$

The unique solution obeying the constraint equation (30) is

$$
\begin{aligned}
&G(k) = k^2 H(k) \\
&G_{ij}(k) = \frac{1}{c}\sqrt{k^2 + \frac{2q^2 v^2 \Delta_0^2}{c^2}}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right) + \frac{q^2}{c^2} H(k)\frac{k_i k_j}{k^2} \\
&G_i(k) = -\mathrm{i}k_i \frac{q}{c} H(k) \\
&H(k) = \frac{\sqrt{2}\Delta_0}{\sqrt{(q^2/c^2) + (k^2/2\Delta_0^2 v^2)}}.
\end{aligned}
\tag{32}
$$

The fact that there exists only one non-degenerate physical ground state in the superconducting phase is very important. Neglecting coupling to electromagnetism, there would have been a degeneracy: shifts of the $\theta(k=0)$ variable. The coupling to electromagnetism imposes the Coulomb constraint eliminating this degeneracy. One can check explicitly that Ψ is an eigenfunction of the electric charge operator $Q \equiv \tilde{\pi}(k=0)$, stressing once again that $U_e(1)$ is unbroken in the superconductor.

The magnetic flux symmetry generator $\hat{\Phi}$ annihillates the vacuum Ψ_0. This can be seen as follows. Due to the fact that $G_{ij}(k=0) \neq 0$ (there are no zero modes), the wavefunctional $A_k^i \Psi_0[\theta, A]$ is normalizable for any k. Therefore

$$
\hat{\Phi}\Psi_0 = \lim_{k\to 0} \epsilon^{ij} k^i A^j(k)\Psi_0 = 0. \tag{33}
$$

Now we turn to the calculation of the expectation value of the order parameter V equations (8)–(11). Using the Gauss law equation (26) and integrating by parts it can be rewritten via operators E_i only

$$
V = \exp\left[\mathrm{i}g\int \tilde{a}_i(x)\hat{E}_i(x)\right] \tag{34}
$$

with $\tilde{a}_i(x) = \delta_{i2}\delta(x_2)\theta(x_1)$ where $\theta(x)$ is a step function. For Gaussian wavefunctionals the expectation value of any operator factorizes [7]

$$
\begin{aligned}
\langle V\rangle &= \exp\left\{-\frac{g^2}{2}\int_{k,l}\langle \tilde{a}^i(k)\hat{E}^i(k)\tilde{a}^j(l)\hat{E}^j(l)\rangle\right\} \\
&= \exp\left\{-\frac{g^2}{2}\int_k \tilde{a}^i(k)\tilde{a}^j(-k)\langle \hat{E}^i(k)\hat{E}^j(-k)\rangle\right\}
\end{aligned}
\tag{35}
$$

where

$$
\begin{aligned}
&\tilde{a}^i(k) \equiv \int \mathrm{d}^2x\, \mathrm{e}^{\mathrm{i}kx}\tilde{a}^i(x) \\
&\tilde{a}^1(k) = 0 \qquad \tilde{a}^2(k) = \mathrm{i}/k_1.
\end{aligned}
\tag{36}
$$

For $\tilde{\Psi}_0$ equations (31)–(32), the correlator is

$$
\langle \hat{E}^i(k)\hat{E}^j(-k)\rangle = G^{ij}(k) \tag{37}
$$

and the result is

$$\langle V\rangle = \exp\left\{-\frac{g^2}{2c}\int \mathrm{d}^2k\,\frac{1}{k_1^2k^2}\left[k_1^2\sqrt{k^2+\frac{2q^2v^2\Delta_0^2}{c^2}}\right.\right.$$
$$\left.\left.+\;k_2^2\sqrt{\frac{2q^2v^2\Delta_0^2}{c^2+(k^2c^4/2\Delta_0^2q^2v^2)}}\right]\right\}. \tag{38}$$

The integration over k_1 diverges linearly in the infrared and we obtain

$$\langle V\rangle = \mathrm{e}^{-aL} \tag{39}$$

where a is a finite scale and L is the sample's dimension (an infrared cutoff). In the $L\to\infty$ limit the VEV vanishes.

4. Three-dimensional case and conclusion

We used the BCS-type theory in two dimensions in order to illustrate the concept of the flux symmetry in the simplest setting. Most superconductors are, however, three-dimensional (although sometimes highly anisotropic). In this section we describe the generalization of the flux symmetry to three dimensions [8].

Here instead of one flux density operator (magnetic field) B, there are three components of a vector $\rho_\Phi^i = B_i$, each of which is a density of a conserved charge. This is the consequence of the homogeneous Maxwell's equations

$$\frac{1}{c}\frac{\partial}{\partial t}B_i = -\epsilon_{ijk}\partial_j E_k.$$

With the identification of $-\epsilon_{ijk}E_k = j_{\Phi j}^i$ as three current densities these are continuity equations ensuring the conservation of three global charges $\Phi_i = \int \mathrm{d}^3x\, B_i(x)$. However out of the three flux charge densities only two are independent, due to the constraint

$$\partial_i B_i = 0.$$

The additional complication compared with two dimensions is that the charge densities themselves are not rotational scalars†.

In the normal state the vacuum is degenerate under the action of Φ_i and the symmetry is spontaneously broken. To convince oneself that this is indeed the case one can either repeat the arguments of section 2 or perform the duality transformation. The latter is performed in the appendix. The photons are the Goldstone modes resulting from this spontaneous breakdown. Two polarizations of massless photons correspond to two independent broken symmetries [8].

In the superconducting state the flux symmetry is unbroken, $B_i|0\rangle = 0$. Correspondingly the spectrum does not contain massless excitations. The open flux tubes carry the conserved quantum number of flux. H_{c1} in three dimensions is determined

† It is a special feature of two dimensions that B is a rotational scalar. This is related to the fact that the photon in two dimensions has only one transverse polarization and is equivalent to scalar particle.

again by the energy density of the flux tube. Now it is much more complicated to construct explicitly the order parameter and at this stage we cannot present an expression which satisfies all the relevant symmetries.

To summarize, we have shown that the superconductor–normal phase transition can be described in the same terms as the standard order–disorder phase transition. The symmetry which is spontaneously broken at the phase transition point is generated by the magnetic flux symmetry. In two dimensions we were able to construct explicitly the local order parameter. Its VEV vanishes in the superconducting phase while it is non-zero in the normal phase. The flux symmetry in the normal phase is spontaneously broken which leads to the appearance of the massless Goldstone mode—the photon. In the superconducting phase the symmetry is unbroken. The existence of the energy gap Δ_Φ in the flux carrying channel is the essence of the phenomenon of perfect diamagnetism (the Meissner effect). This energy gap (which determines the critical magnetic field H_{c1}) is more directly related to the perfect diamagnetism than the gap Δ_e in the electrically charged channel†.

These conclusions are independent of the particular microscopic model of superconductivity. For example, they apply to novel hypothetical mechanisms of superconductivity which appeared in connection to high T_c materials like two-dimensional Hubbard models, anyon superconductivity etc.

We hope that this new general point of view provides additional insight into the phenomenon of superconductivity and gives a more satisfactory universal description of the phase transition in gauge-invariant physical terms.

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Appendix

In this appendix we perform dual transformation in the free Maxwell theory. In two dimensions this leads to a theory of a free massless scalar. In three dimensions this is a theory of a free dual vector potential. In both cases the magnetic flux symmetry in dual formulation is represented by global shift of a massless field and its spontaneous breaking is apparent on the classical level. We use a variant of duality transformation introduced in [9].

Let us start with two dimensions. Canonically (in Hamilton gauge) the theory is described by the Hamiltonian

$$H = \frac{1}{2}\int \mathrm{d}^2x\,(E^2 + B^2) \tag{A1}$$

with the constraint $\partial_i E_i = 0$ The linear constraint is easily solved in terms of a single scalar field $\chi(x)$

$$E_i = \epsilon_{ij}\partial_j\chi. \tag{A2}$$

† Indeed there exist gapless superconductors with $\Delta_e = 0$ which nevertheless exhibit the Meissner effect.

The magnetic field $B(x)$ is just the momentum canonically conjugate to $\chi(x)$

$$B = -\pi. \tag{A3}$$

In these variables the Hamiltonian becomes

$$H = \frac{1}{2}\int \mathrm{d}^2x\,[\pi^2 + (\partial_i\chi)^2]. \tag{A4}$$

The flux symmetry in these notations is the familiar shift transformation

$$\chi(x) \to \chi(x) + \text{constant}. \tag{A5}$$

This symmetry is spontaneously broken [3], with χ itself (which interpolates the photon) as a Goldstone boson.

This statement remains true in the normal (Coulomb) phase of the interacting theory as well, although one now can not explicitly solve the Coulomb constraint [5].

In three-dimensional QED the magnetic symmetry is more complicated. The current† $\tilde{F}_{\mu\nu}$ is an antisymmetric tensor and hence the corresponding charges $\Phi_\mu = \int \mathrm{d}^3x\,\tilde{F}_{\mu 0}$ are no longer rotational scalars. Φ_0 identically vanishes, while out of three charge densities $\tilde{F}_{0i}(x) = B_i(x)$ only two are independent: $\partial_i B_i(x) = 0$. Spontaneous breakdown of these two symmetries leads to the appearance of two massless photons with helicities ± 1. In the free Maxwell theory one still has a 'shift' interpretation of the magnetic symmetry in terms of the dual vector potential' $\chi_i(x)$. The Coulomb constraint is solved by

$$E_i = \epsilon_{ijk}\partial_j\lambda_k. \tag{A6}$$

The magnetic field B_i is still conjugate to λ_i in Hamiltonian formalism

$$B_i = -\pi_i \qquad [\lambda_i(x), \pi_j(y)] = \mathrm{i}\delta_{ij}\delta^3(x-y). \tag{A7}$$

In variables χ_i and π_i the Hamiltonian takes a form

$$H = \tfrac{1}{2}\int \mathrm{d}^3x\,\left[\pi_i^2 - \lambda_i(\delta_{ij}\partial^2 - \partial_i\partial_j)\chi_j\right]. \tag{A8}$$

It has a dual gauge symmetry generated by $\partial_i B_i$. The magnetic symmetry generators Φ_i just shift fields χ_i

$$\chi_i(x) \to \chi_i(x) + c_i. \tag{A9}$$

On the physical subspace, however, they are not independent, since $\partial_i B_i = 0$. Two transversal components of χ_i interpolate physical photons. In accordance with the Goldstone theorem they are the Goldstone bosons corresponding to the breakdown of two independent generators of the flux symmetry.

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